## Errata

## Chapter 1

## Matrices

### 1.1 The basic operations

October 7, 2021
Math 1.1.1. Page 7: Denoting inverse of $A$ by $A^{-1}$ should be deferred until proving its uniqueness.

### 1.4 Determinants

Math 1.4.1. Page 20: In the last paragraph, in "... except for those in the rows indexed by $k$. ., "rows" must be replaced by the singular "row". Also mention that the other rows of $D$ (that is, those not indexed by $k$ ) are equal to the corresponding rows of $A$ (and hence $B$ ).

Also, in (1.4.8), replace the index $i$ with $k$ to maintain consistency.
Math 1.4.2. Page 23: The submatrices are wrongly referred to as minors at various places.

## Exercises

October 27, 2021

Math 1.1. Page 32: In Exercise 1.13, it'll be cleaner (and still equivalent) to replace $I+A$ with $I-A$.
Math 1.2. Page 32: In Exercise 1.15, $e_{j}$ in $e_{j} A e_{k}$ should be replaced by $e_{j}^{T}$.
Math 1.3. Page 34: In Exercise 5.4, the boldface is confusing. Omit it.
Math 1.4. Page 35: In Exercise M. 7 (c), $p(t)$ is of degree at most $n$ and not necessarily $n$.

## Chapter 2

## Groups

### 2.1 Laws of composition

October 28, 2021
Math 2.1.1. Page 39: In Proposition 2.1.4, condition (ii) is implied by (iii) and can (so, must) be dropped.

### 2.4 Cyclic groups

October 29, 2021
Math 2.4.1. Page 46: In Proposition 2.4.2 (b) $r \geq s$ is redundant.

### 2.8 Cosets

November 4, 2021
Math 2.8.1. Page 58: In the paragraph after Corollary 2.8.13, mention that $n \geq 2$.

Math 2.8.2. Page 58: The proof of Proposition 2.8.14 is wrong/incomplete: That $g_{i} h_{1} K, \ldots, g_{i} h_{n} K$ will be a partition of $K$ follows not due to the fact that "multiplication by $g_{i}$ is an invertible operation". instead, it follows since $K \subseteq H$ and that group multiplication is associative.

Also, it should be noted that $g_{i} h_{j} K$ actually stands for $\left(g_{i} h_{j}\right) K$.
Math 2.8.3. Page 59: The notation $g \mathrm{Hg}^{-1}$ should be explained (or refer to Exercise 6.7, where it is explained): It is not a coset (since $g H$ or $\mathrm{Hg}^{-1}$ may fail to form subgroups). Instead, it is the set $\left\{g h g^{-1}: h \in H\right\}$.

Math 2.8.4. Page 59: In Proposition 2.8.18 (a), it is used that the image of a subgroup under a homomorphism is a homomorphism, but never proved before.

### 2.10 The correspondence theorem

November 9, 2021
Math 2.10.1. Page 62: In Example 2.10.3, $n$ 's in "...such as an $n$-cycle with $n$ odd. ..." must be replaced by $k$ (or some other symbol which should not be $n$ ).

### 2.11 Product groups

November 11, 2021
Non-math 2.11.1. Page 64: The second paragraph has a missing period in the sentence before Figure (2.11.2).

Math 2.11.2. Page 64: In second paragraph, the 1 's in $G \times 1$ and $1 \times G^{\prime}$ should be replaced with $\{1\}$.

### 2.12 Quotient groups

November 11, 2021
Math 2.12.1. Page 66: The consistency of notation $\mathbb{Z} / \mathbb{Z} n$ with that introduced for Quotient groups should be checked. (Similarly, the consistency of $a+\mathbb{Z} n$ with the definition of cosets given later should be checked.)

## Exercises

November 13, 2021
Math 2.1. Page 70: In Exercise 4.2, mention to use that the $n$-th roots of unity are $\left\{e^{i 2 \pi \theta / n}\right\}_{\theta=0}^{n-1}$.
Math 2.2. Page 70: In Exercise 4.7, mention $x, y$ and 1 to be all distinct.
November 19, 2021
Math 2.3. Page 73: In Exercise 8.10, mention that cosets of $H$ are to be found in $G$, and not in $G L_{2}(\mathbb{R})$.

Math 2.4. Page 73: In Exercise 8.12, $a S$ is called a coset even though $S$ is not a subgroup. The text defined cosets only fr subgroups.

Math 2.5. Page 73: In Exercise 9.3, mention $a$ to be a positive integer.
Math 2.6. Page 74: In Exercise 10.3, mention that the existence of such a $\phi$ is possible since 6 divides 12 .

Math 2.7. Page 77: In Exercise M.11, (b), mention explicitly that we are talking od only invertible matrices. The decomposition is not unique for non-invertible matrices. (See this.)

## Chapter 3

## Vector spaces

November 20, 2021

### 3.1 Subspaces of $\mathbb{R}^{n}$

Math 3.1.1. Page 79: In (3.1.3), use $k$ instead of $n$ since $n$ is already used in stating that $W$ is a subspace of $\mathbb{R}^{n}$.

Non-math 3.1.2. Page 79: In checking that the nullspace is a subspace, in the second bullet point, $c X$ in " $A c X$ " should be enclosed in parentheses.

Math 3.1.3. Page 79: In the proof of Proposition 3.1.4, the proof of the second part is presented before the first, which it implicitly uses.

Math 3.1.4. Page 81: In Definition 3.2 .2 (iii) third bullet point, remark the precedence convention for multiplication and addition so that $a b+a c$ means $(a b)+(a c)$.

### 3.2 Fields

November 22, 2021
Math 3.2.1. Page 84: In the proof of Lemma 3.2.10, it is implicitly assumed that $\underbrace{1+\cdots+1}_{r s \text { times }}=(\underbrace{1+\cdots+1}_{r \text { times }}) \times(\underbrace{1+\cdots+1}_{s \text { times }})$. It is unnoticed because of
the usage of the familiar bar notation for equivalence classes, in which it is true that $\bar{r} \bar{s}=\overline{r s}$. But it is not a priori true here. It must be stated as an exercise or a separate lemma.

Non-math 3.2.2. Page 85: In the paragraph after Example 3.3.4, the "bijective map" in ". . . bijective map such that. .." should be omitted.

### 3.5 Computing with bases

## December 6, 2021

Math 3.5.1. Page 94: In the $(\cos t, \sin t)$ example of basechange computaion, remark that the basechange matrix has to be a matrix of complex numbers and not functions (in which case, a general matrix $\left[\begin{array}{cc}1+(i-x) \tan t & 1-(i+y) \tan t \\ x & y\end{array}\right]$ with $x, y \in \mathbb{C}$ would suffice).

## Exercises

December 11, 2021
Non-math 3.1. The first section in the text is Subspaces of $\mathbb{R}^{n}$. But the Exercises enlist the Fields to be the first section. Fix this.

December 24, 2021
Math 3.2. Page 100: Exercise 5.1 can be generalized for any field (and not just $\mathbb{R}$ ) with characteristic not equal to 2 .
Math 3.3. Page 100: In Exercise 5.3, 0's should be replaced with $\{0\}$ 's.
Math 3.4. Page 101: In Exercise 6.3, replace "For every positive integer, we..." with "For every positive integer $p$, we...".

December 29, 2021
Math 3.5. Page 101: In Exercise M.2, the coefficient $c_{0}$ in the polynomial equation should be accompanied by the indentity matrix $I$.

## Chapter 4

## Linear Operators

### 4.1 The matrix of a linear transformation

January 3, 2022
Math 4.1.1. Page 105: In Proposition 4.2.5, the order of quantifier should be revised. It states that for any $X_{n \times 1}$ and any $Y_{n \times 1}$, there exists a matrix $A$. Rather, existence should be outside with the double universal quantification nested inside it.

### 4.3 Linear operators

January 6, 2022
Math 4.3.1. After the proof for Proposition 4.3.1, also remark that (b) is not necessarily true for infinite-dimensional spaces. (The same examples as given for the similar comment afout (a) will suffice.)

## Appendix

December 29, 2021
Math 4.1. Page 514: In A.1.2 (ii), " $k>1 "$ should be " $k \geq 1$ ". Also, in the following line, " $k \geq 0$ " should be replaced by " $k \geq 1$ " (since the sentences $P_{k}$ are indexed by $\left.n \in \mathbb{N}=\{1,2,3, \ldots\}\right)$.

Math 4.2. Page 515: The line "So a proof using complete induction must include a proof of $P_{1}$." must be omitted - if the statement that "If $n$ is a positive integer and $P_{k}$ is true for every positive integer $k<n$, then $P_{n}$ is true." is already proven, then no need to explicitly show $P_{1}$ 's proof.

