

# Errata

# Chapter 2

## 2.1

*April 22, 2021*

**Non-math 2.1.1.** Add ‘Vol I’ or ‘Vol II’ to the references.

*April 23, 2021*

**Non-math 2.1.2.** Prettier way to write the  $C$  programming language.

**Math 2.1.3.** Proposition 2.3.9 should be called Euclid’s division lemma, and not Euclidean algorithm.

# Chapter 3

## 3.3

*May 9, 2021*

**Math 3.3.1.** In Definition 3.3.1, “Then the *function*  $f: X \rightarrow Y\dots$ ” should be “...a *function*...”, since the equality for functions is not yet defined (or axiomatized).

**Math 3.3.2.** In Definition 3.3.1’s second to last sentence, “the unique object  $f(x)$  for which  $P(x, f(x))$  is true” should read “the unique object  $f(x) \in Y$  for which  $P(x, f(x))$  is true”.

Similarly, in the second line of Example 3.3.2, “there is exactly one  $y$  for which” should read “there is exactly one  $y \in \mathbb{N}$  for which”.

**Math 3.3.3.** Definition 3.3.7 should be elevated to the status of an axiom, because of the comments I posted as Atom dated 16 May, 2021 at 11:34 am. This makes the first part of Exercise 3.3.1 redundant.

**Non-math 3.3.4.** In Remark 3.3.25, the last line should refer to Example 3.3.22 and not to “previous example”.

**Math 3.3.5.** In Exercise 3.3.8, (d) can be generalized by weakening the assumption  $X \cap Y = \emptyset$  to the assumption that for each  $x \in X \cap Y$ ,  $f(x) = g(x)$ .

## 3.4

*May 10, 2021*

**Math 3.4.1.** In Definition 3.4.1, “ $S$  is a set in  $X$ ” should be replaced with “ $S \subseteq X$ ”.

**Math 3.4.2.** In Definition 3.4.1, better notation would be  $f[S]$  (what if both  $S \in X$  and  $S \subseteq X$  for  $f: X \rightarrow Y$ ?).

**Math 3.4.3.** Axiom 3.11 should be renamed to “Function set axiom”.

**Math 3.4.4.** In Lemma 3.4.10, “the set  $\{Y : Y \text{ is a subset of } X\}$  is a set” should be replaced by “there exists a set  $Z$  such that for any object  $Y$ , we have  $Y \in Z \iff Y \subseteq X$ ”.

Similar remarks to all the instances when some expression  $\{\dots\}$  is claimed to be a set. Among these are unions and intersections of families of sets on pages 60-61.

*May 11, 2021*

**Non-math 3.4.5.** In Exercise 3.4.11, ‘de Morgan’ should be ‘De Morgan’, just like in the main text.

## 3.5

*May 16, 2021*

**Math 3.5.1.** The definitions of equality of ordered pairs and ordered  $n$ -tuples should both be elevated to the status of axiom, since the original definitions are not sufficient to ensure that they determine their components uniquely. (Similar to my comments as Atom on the main site regarding axiomatization of function equality, dated 16 May, 2021 at 11:34 am.) After this Exercise 3.5.3 becomes redundant.

**Math 3.5.2.** In the erratum for the third edition (hardback) that mentions to replace Remark 3.5.5, and the erratum about Exercise 3.5.1, Math 3.4.4 should be applied.

**Math 3.5.3.** In the paragraph after Example 3.5.6, it should be added that  $f(x, y)$  is short for  $f((x, y))$ . The former should be strictly used when  $f$  is a function on two variables, while the latter, when  $f$  is a function of one variable with domain as some Cartesian product.

**Math 3.5.4.** It should be remarked that we don't require the concept of ordered  $n$ -tuple in Exercise 3.5.12, which otherwise would have been circular (since the definition ordered  $n$ -tuples required the set  $\{1, \dots, n\}$ ). Note that, as stated, the exercise does require the concept of an ordered pair, which in turn doesn't require anything from the naturals.

**Math 3.5.5.** In Example 3.5.9, it should be remarked that the ordered triples are defined in an analogous way to ordered pairs (since they are not defined yet and one needs them for this example).

**Math 3.5.6.** In Remark 3.5.10,  $x_1, \dots, x_n$  should be  $(x_1, \dots, x_n)$ .

**Math 3.5.7.** In the paragraph after Remark 3.5.11, replace ‘ $X^2$  is the Cartesian product  $X \times X$ ’ by “ $X^2$  is identifiable with the Cartesian product  $X \times X$ ”.

*May 17, 2021*

**Math 3.5.8.** In Exercise 3.5.6, the assumption  $A, B, C, D$  being nonempty can be weakened to just  $A, B$  being nonempty.

**Math 3.5.9.** In Exercise 3.5.7, why compare with Exercise 3.1.7?

**Math 3.5.10.** In Exercise 3.5.9, also add a second part with intersections and unions interchanged.

*May 18, 2021*

**Math 3.5.11.** Just like ordered pairs and ordered pairs can be defined via sets using Exercises 3.5.1 and 3.5.2, add an exercise that defines functions in terms of sets (as was promised in Remark 3.3.10).

## 3.6

*May 30, 2021*

**Math 3.6.1.** In Remark 3.6.3, even naturals need to be first defined.

*June 2, 2021*

**Math 3.6.2.** Define unbounded sets for Remark 3.6.13.

# Chapter 4

## 4.1

*June 17, 2021*

**Math 4.1.1.** In definition 4.1.1, comment that the existence of the set  $\mathbb{Z}$  follows from two applications of replacement. Similar for Definition 4.2.1 for  $\mathbb{Q}$ .

## 4.3

*June 22, 2021*

**Math 4.3.1.** Exercise 4.3.5 can be strengthened to have  $2^N > N$  instead of  $2^N \geq N$ .

*June 23, 2021*

**Math 4.3.2.** In Proposition 4.3.7 (g), the condition “ $z$  is non-zero” can be relaxed.

## 4.4

*June 23, 2021*

**Math 4.4.1.** The way the proof of Proposition 4.4.4 is presented, it is not explicitly clear that axiom of choice need *not* be evoked. So it should be made clear, adding a comment on the construction of the infinite sequence of the  $p$ 's.

# Chapter 5

## 5.1

*June 25, 2021*

**Non-math 5.1.1.** In Remark 5.1.9, delete “(Proposition 6.1.4)”.

**Non-math 5.1.2.** In Remark 5.2.4, “notations” should be replaced with “definitions”.

## 5.3

*June 27, 2021*

**Non-math 5.3.1.** In the paragraph after the proof of Proposition 5.3.11, “On obvious...” should be “One obvious...”.

## 5.4

*June 30, 2021*

**Math 5.4.1.** The title of Proposition 5.4.9 should not be that non-negative reals are closed since we are not yet considering Cauchy sequences of reals.

## 5.5

*July 8, 2021*

**Math 5.5.1.** The opening paragraph should be corrected since one can't take least upper bound for *any* subset of  $\mathbb{R}$ .

**Math 5.5.2.** In the third paragraph of the proof of Theorem 5.5.9, the reason for existence of an integer  $L$  such that  $L/n < x_0$  is *not* the Archimedean property.

*July 10, 2021*

**Math 5.5.3.** In Remark 5.5.11's last line, enclose  $-\infty$  in parentheses.

**Math 5.5.4.** In the hint for Exercise 5.5.4, Exercise 5.4.8 is an overkill; just use Corollary 5.4.10.

*July 11, 2021*

**Math 5.5.5.** For Lemma 5.6.6 (a) to make sense, it must be ensured that  $y \neq 0$  since exponentiation of zero by integers (and therefore by positive integers) is not defined. Even after this, (c) should be proved prior to (a).

## 5.6

*July 25, 2021*

**Non-math 5.6.1.** In title of Definition 6.3.1, "Sup" and "inf" should be written via `\sup` and `\inf`.



# Chapter 6

## 6.1

*July 13, 2021*

**Math 6.1.1.** In the first paragraph of section 6.1, it's written “we never really finished the job of constructing the real numbers, because we never got around to replacing formal limits with actual limits”. But the real numbers *have* been constructed, only that we have not yet defined actual limits. But that does not mean that they haven't been *constructed* fully.

**Non-math 6.1.2.** Proposition 6.1.15: To maintain consistency with LIM and  $\lim$ , make LIM an starred operator in LaTeX so that  $n \rightarrow \infty$  appears below LIM rather than in subscript.

**Math 6.1.3.** In Exercise 6.1.6, the  $m$  in  $(a_n)_{n=m}^{\infty}$  should be 1 since LIM is defined only for Cauchy sequences of rationals which start from index 1. Also, write LIM in math mode.

**Math 6.1.4.** In Theorem 6.1.19 (g) and (f), use of max and min is made, which are not yet defined.

*July 19, 2021*

**Math 6.1.5.** Add axioms postulating the existstence of  $+\infty$  and  $-\infty$ .

*July 26, 2021*

**Math 6.1.6.** The sequences defined in Exercise 6.1.1 are referred to as “increasing sequences”, which is in collision with the “increasing sequences” as defined in Proposition 6.3.8. This can be prevented by calling the former as “strictly increasing sequences”.

## 6.4

*July 27, 2021*

**Non-math 6.4.1.** Missing ellipsis in the  $-1/2, -1/2, -1/4, -1/4, -1/6, -1/6$  sequence of Example 6.4.9.

*July 30, 2021*

**Math 6.4.2.** The second line of the paragraph after the proof of Proposition 6.4.12 holds if  $L^+, L^- \in \mathbb{R}$ , their equality is not enough.

**Math 6.4.3.** Squeeze test could be made stronger by requiring  $\min(a_n, c_n) \leq b_n \leq \max(a_n, c_n)$  instead of  $a_n \leq b_n \leq c_n$  for each  $n$ .

## 6.6

*August 6, 2021*

**Math 6.6.1.** In Exercise 6.6.3, it will be helpful to write that the well-ordering principle follows from the already proven principle of infinite descent.

# Chapter 7

## 7.1

*August 31, 2021*

**Math 7.1.1.** In Remark 7.1.10, it should be mentioned that  $\{y \in X : P(y)\}$  is a finite set.

**Math 7.1.2.** Lemma 7.1.4:  $m - 1 \leq n \leq p$ .

# Appendix A

## A.1

*April 23, 2021*

**Non-math A.1.1.** “ $4 = 4$  is more precise”. Replace ‘precise’ with ‘efficient’.

*April 24, 2021*

**Non-math A.1.2.** Wrong periods in the last line of the second paragraph of Negation.

## A.5

**Non-math A.5.1.** In Remark A.5.1, ‘Aristotlean’ should be ‘Aristotelian’.